

# Joint Generative-Discriminative Aggregation Model for Multi-Option Crowd Labels

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## Motivations

- Crowd labels are often *noisy* and unreliable, since crowd workers are usually *inexpert* in the assigned tasks.
- A *crowdsourcing aggregation model* is required to estimate the true labels by aggregating the redundant crowd labels.
- Recent studies have shown that crowd workers cannot completely convey their *non-deterministic* beliefs with the *single-option* crowd labels.
- The standard aggregation models are often incompatible with the *multi-option* crowd labels, and they are only able to handle single-option crowd data.

## Contributions

- Proposing a new *discriminative* aggregation model with *convex* problem, and deriving an efficient *optimization* algorithm to solve the corresponding problem.
- Introducing a novel *joint generative-discriminative* aggregation model, coupled via a probabilistic framework.
- Achieving superior or competitive results on *single-option* and *multi-option* crowdsourcing datasets with much *faster running speed* compared to alternative models.

## Discriminative Aggregation Model

Objective function:

$$\min_{\mathbf{w}, \mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1} \sum_{i=1}^N \|\mathbf{X}_i \mathbf{w} - \mathbf{y}_i\|_1 + \lambda_w \|\mathbf{w}\|_2^2$$

Approximation:

$$\min_{\mathbf{w}, \mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1} \sum_{i=1}^N \mathbf{w}^T (\mathbf{X}_i^T \mathbf{U}_i \mathbf{X}_i + \lambda_w \mathbf{I}) \mathbf{w} - 2\mathbf{y}_i^T \mathbf{U}_i \mathbf{X}_i \mathbf{w} + \mathbf{y}_i^T \mathbf{U}_i \mathbf{y}_i$$

Update  $\mathbf{w}$ :

$$\min_{\mathbf{w}} \sum_{i=1}^N \mathbf{w}^T (\mathbf{X}_i^T \mathbf{U}_i \mathbf{X}_i + \lambda_w \mathbf{I}) \mathbf{w} - 2\mathbf{y}_i^T \mathbf{U}_i \mathbf{X}_i \mathbf{w}$$

Update  $\mathbf{Y}$ :

$$\min_{\mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1} \sum_{i=1}^N \mathbf{y}_i^T \mathbf{U}_i \mathbf{y}_i - 2\mathbf{y}_i^T \mathbf{U}_i \mathbf{X}_i \mathbf{w}$$

**Algorithm 1:** CWMV<sub>ℓ1</sub> aggregation model

- 1 Initialize  $\mathbf{Y}$  by majority voting
- 2 **while not converged do**
- 3  $\mathbf{w} = \left( \sum_{i=1}^N \mathbf{X}_i^T \mathbf{U}_i \mathbf{X}_i + \lambda_w \mathbf{I} \right)^{-1} \left( \sum_{i=1}^N \mathbf{X}_i^T \mathbf{U}_i \mathbf{y}_i \right)$
- 4  $\min_{\mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1} \mathbf{y}_i^T \mathbf{U}_i \mathbf{y}_i - 2\mathbf{y}_i^T \mathbf{U}_i \mathbf{X}_i \mathbf{w} \quad \forall i \in \{1, \dots, N\}$
- 5 **end**

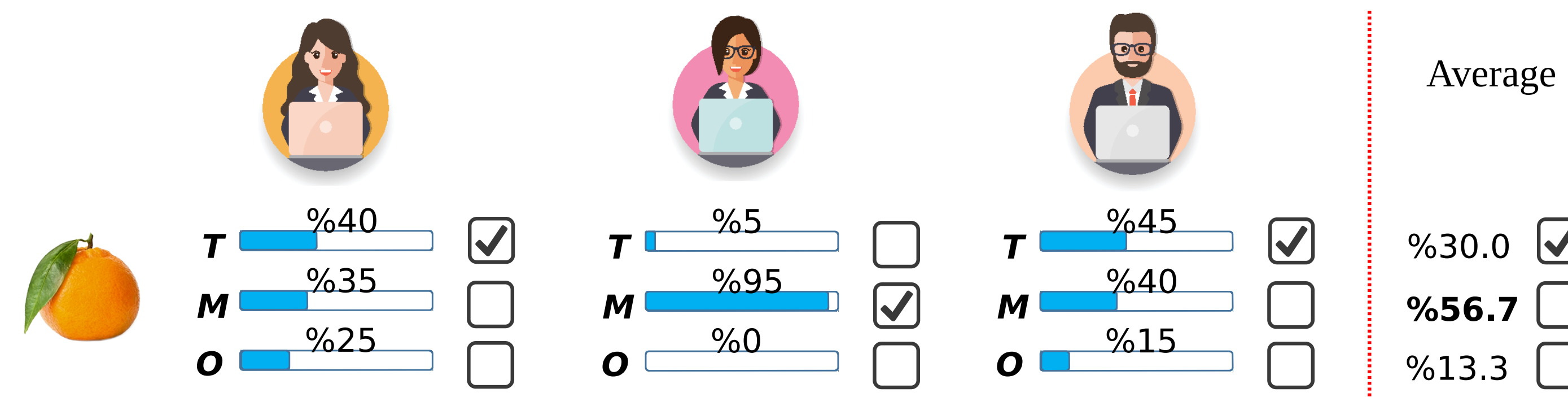


Figure 1: Three crowd workers are asked to classify a figure as tangor (T), mandarin (M) or orange (O). Their single-option and multi-option crowd labels are shown with checked boxes and confidence bars respectively. The average score of multi-option labels correctly shows higher chance for mandarin, while the majority of single-option labels incorrectly suggests tangor as the truth.

## Joint Generative-Discriminative Aggregation Model

Objective function:

$$\min_{\mathbf{w}, \mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1, \nu_j} \mathbf{KL}(p^\nu \| p_0^\nu) - \sum_{ijk} x_{ijk} y_{ic} \log(\nu_{jck}) + \gamma \sum_i \|\mathbf{X}_i \mathbf{w} - \mathbf{y}_i\|_1 + \lambda_w \|\mathbf{w}\|_2^2$$

Approximation:

$$\min_{\mathbf{w}, \mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1, \nu_j} \mathbf{KL}(p^\nu \| p_0^\nu) - \sum_{ijk} x_{ijk} y_{ic} \log(\nu_{jck}) + \sum_i \mathbf{w}^T (\gamma \mathbf{X}_i^T \mathbf{U}_i \mathbf{X}_i + \lambda_w \mathbf{I}) \mathbf{w} - 2\gamma \mathbf{y}_i^T \mathbf{U}_i \mathbf{X}_i \mathbf{w} + \gamma \mathbf{y}_i^T \mathbf{U}_i \mathbf{y}_i$$

Update  $\mathbf{Y}$ :

$$\min_{\mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1} - \sum_{ijk} x_{ijk} y_{ic} \log(\nu_{jck}) + \sum_i \mathbf{w}^T (\gamma \mathbf{X}_i^T \mathbf{U}_i \mathbf{X}_i + \lambda_w \mathbf{I}) \mathbf{w} - 2\gamma \mathbf{y}_i^T \mathbf{U}_i \mathbf{X}_i \mathbf{w} + \gamma \mathbf{y}_i^T \mathbf{U}_i \mathbf{y}_i$$

Update  $\nu$ :

$$\min_{\nu_j} \mathbf{KL}(p^\nu \| p_0^\nu) - \sum_{ijk} x_{ijk} y_{ic} \log(\nu_{jck})$$

Update  $\mathbf{w}$ :

$$\min_{\mathbf{w}} \sum_{i=1}^N \mathbf{w}^T (\gamma \mathbf{X}_i^T \mathbf{U}_i \mathbf{X}_i + \lambda_w \mathbf{I}) \mathbf{w} - 2\gamma \mathbf{y}_i^T \mathbf{U}_i \mathbf{X}_i \mathbf{w}$$

**Algorithm 2:** DS-CWMV<sub>ℓ1</sub> aggregation model

- 1 Initialize  $\mathbf{Y}$  by majority voting
- 2 **while not converged do**
- 3  $\mathbf{w} = \left( \sum_{i=1}^N \mathbf{X}_i^T \mathbf{U}_i \mathbf{X}_i + \frac{\lambda_w}{\gamma} \mathbf{I} \right)^{-1} \left( \sum_{i=1}^N \mathbf{X}_i^T \mathbf{U}_i \mathbf{y}_i \right)$
- 4  $p^{jck} = \text{Dir}(\mu + \sum_i x_{ijk} y_{ic}) \quad \forall j \in \{1, \dots, M\}, \forall \{c, k\} \in \{1, \dots, B\}$
- 5  $\min_{\mathbf{y}_i \geq 0, \mathbf{1}^T \mathbf{y}_i = 1} \mathbf{y}_i^T \mathbf{U}_i \mathbf{y}_i - 2\mathbf{y}_i^T (\mathbf{U}_i \mathbf{X}_i \mathbf{w} + \frac{1}{\gamma} \sum_{jk} x_{ijk} \log(\nu_{jck})) \quad \forall i \in \{1, \dots, N\}$
- 6 **end**

Model	Web Search	Age	RTE	Temp	Flowers	Average
<i>baselines</i>						
MV	26.90	34.88	10.31	6.39	22.00	28.83
IWMV	15.04	34.53	8.12	5.84	19.00	17.09
M <sup>3</sup> V	12.74	33.33	7.88	6.06	13.50	15.43
DS	16.92	39.62	7.25	5.84	13.00	18.69
DS+Prior	13.26	34.53	[7.13]	5.84	13.50	15.80
GLAD	19.30	35.73	<b>7.00</b>	[5.63]	13.50	16.20
Entropy (M)	11.10	<b>31.14</b>	7.50	[5.63]	13.00	14.03
Entropy (O)	10.40	37.32	-	-	-	17.76
CrowdSVM	9.42	33.33	7.75	[5.63]	13.50	13.65
G-CrowdSVM	7.99±0.26	32.98±0.36	7.67±0.19	5.71±0.33	[12.10±1.07]	12.78±0.31
<i>ours</i>						
CWMV <sub>ℓ2</sub>	10.89	34.43	7.25	[5.63]	16.00	14.65
CWMV <sub>ℓ1</sub>	10.70	34.23	7.50	[5.63]	13.00	14.43
DS-CWMV <sub>ℓ2</sub>	[7.58]	32.04	[7.13]	[5.63]	13.00	[12.32]
DS-CWMV <sub>ℓ1</sub>	<b>6.78</b>	[31.54]	<b>7.00</b>	<b>5.41</b>	<b>10.00</b>	<b>11.65</b>

Table 1: Error rates (%) of aggregation models on single-option crowdsourcing datasets.

Model	A-Flag	C-Flag	A-Dog	C-Dog	Average
<i>baselines</i>					
(soft) MV	21.67	20.83	8.98	8.98	12.90
(soft) DS	22.50	20.83	10.16	9.76	13.70
(soft) DS+Prior	20.00	19.17	8.98	8.59	12.23
(soft) Entropy (M)	17.50	16.67	12.89	12.89	14.23
<i>ours</i>					
CWMV <sub>ℓ2</sub>	16.67	16.67	8.98	8.59	11.30
CWMV <sub>ℓ1</sub>	[11.67]	[10.83]	[8.59]	[8.20]	[9.31]
DS-CWMV <sub>ℓ2</sub>	14.17	14.17	9.38	8.59	10.64
DS-CWMV <sub>ℓ1</sub>	<b>13.33</b>	<b>10.00</b>	<b>8.20</b>	<b>7.81</b>	<b>9.17</b>

Table 2: Error rates (%) of aggregation models applied on multi-option crowd datasets.

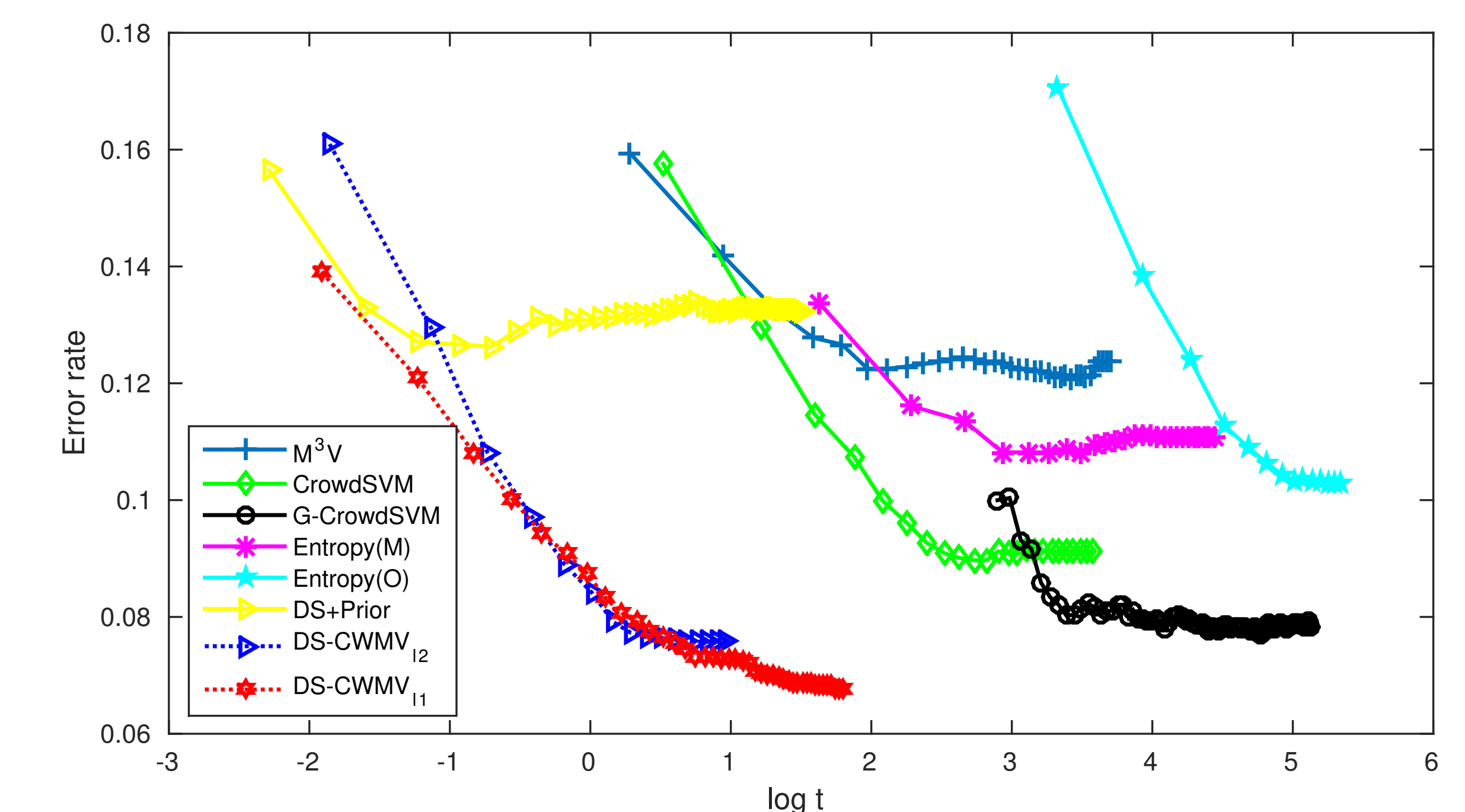


Figure 2: Convergence comparison of aggregation models on Web Search dataset.

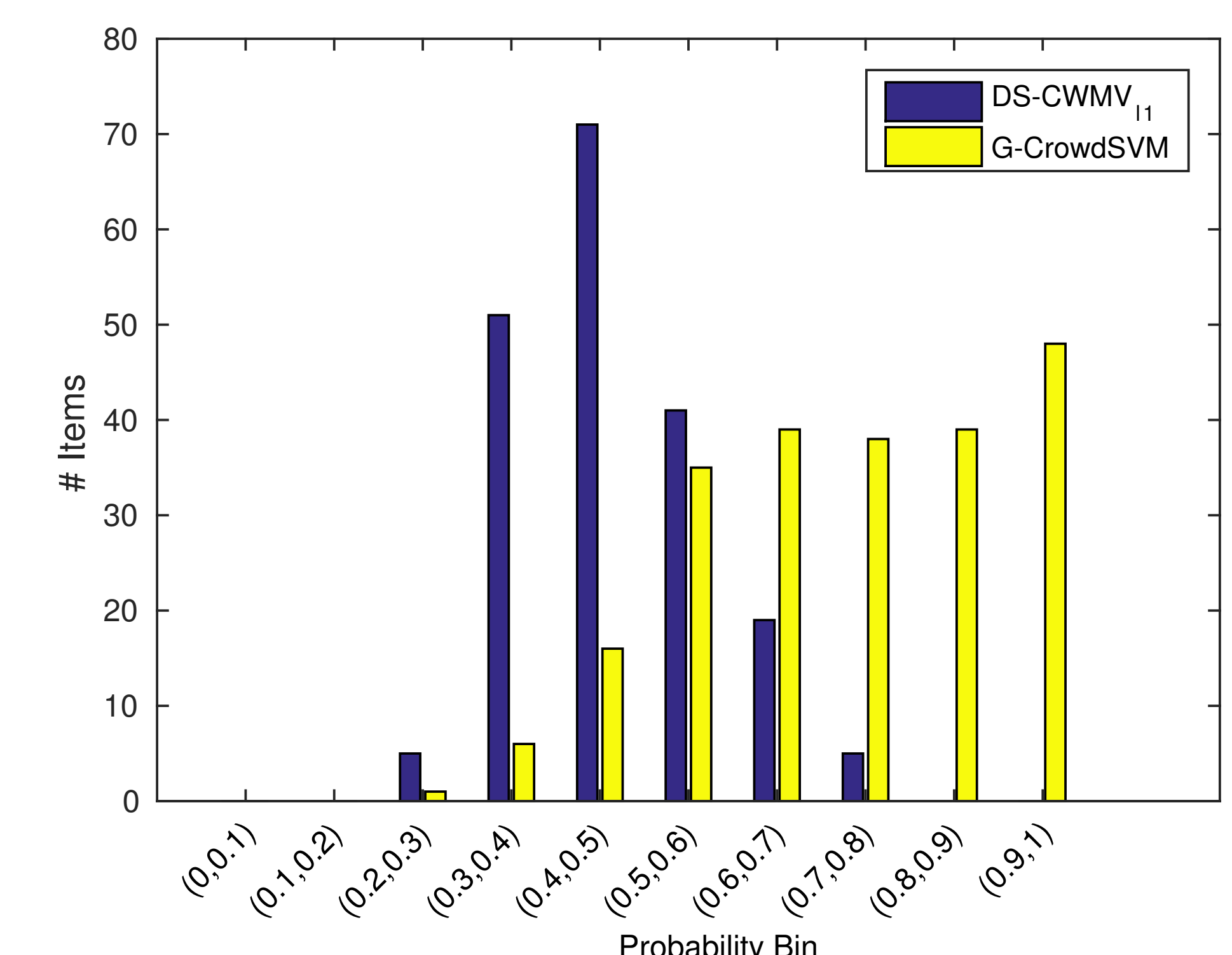


Figure 3: Histogram of the truths for mispredicted items. The results belong to DS-CWMV<sub>ℓ1</sub> and G-CrowdSVM.